

## The example of dynamic optimization decision making in fatigue testing

Tero Frondelius<sup>\*#</sup>, Joonas Vaara<sup>#</sup>, Markus Hartikainen<sup>†</sup>, Karthik Sindhya<sup>†</sup>, and Juho Könnö<sup>\*</sup>

<sup>\*</sup> University of Oulu  
Central of Machine and Vehicle Design  
Pentti Kaiteran katu 1, 90570 Oulu, Finland  
e-mail: firstname.lastname@oulu.fi

<sup>#</sup> Wärtsilä Finland  
R&D and Engineering  
Reininkatu 3, 65170 Vaasa, Finland  
e-mail: firstname.lastname@wartsila.com

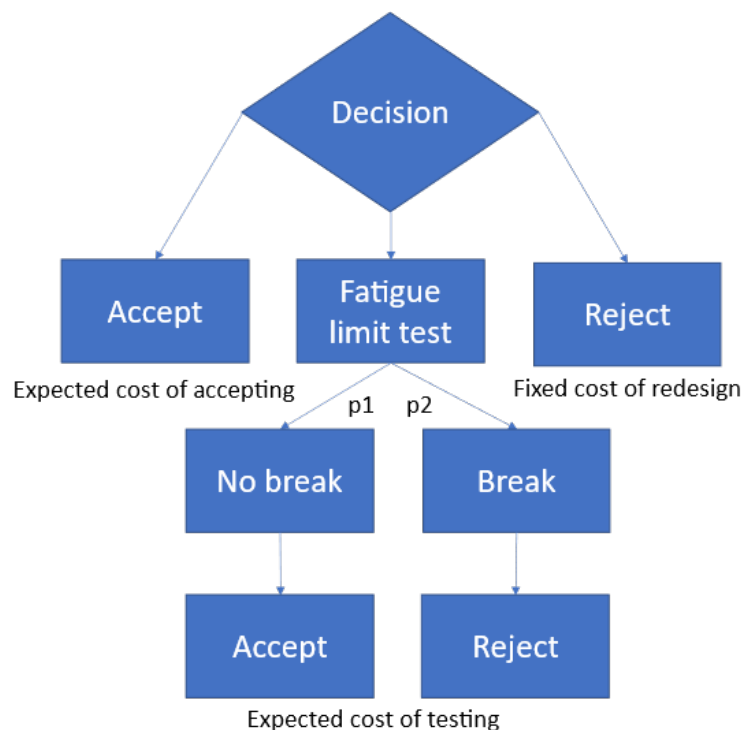
<sup>†</sup> Silo AI, 5th Floor  
Fredrikinkatu 57 C, 00100 Helsinki, Finland  
e-mail: firstname.lastname@silo.ai

### ABSTRACT

The problem of optimal stopping experiments with application to fatigue testing is studied. In order to apply dynamic optimization techniques, the monetary consequences of being wrong when performing fatigue assessment of a machine component with known fatigue loading need to be estimated. [3] The option of redesigning the machine component also exists in the actual design process if it is too risky to accept to the field, see Figure 1. Performing more tests also has monetary costs attached to it. One can divide the example problem into smaller but recursive sub-problems – formally known as the Bellman equation. A roll-out strategy for designing the experiments by either staircase or sequential D-optimal testing [5] is applied to simplify the problem.

We solve the problem with finite-horizon backward induction. The numerical results are promising in minimizing the expected costs due to decision-making. The method approaches the decision to accept the design conservatively, requiring enough evidence from the tests to back up the decision. On the other hand, if the fatigue test results do not seem promising, the decision to redesign the component is reached quickly. [2] One can understand this with the dynamics of Bayesian update of the fatigue strength distribution parameters. Given the failures at relatively low-stress levels, the variance of fatigue strength tends to high values. It requires plenty of evidence of higher fatigue strength to diminish the effect of the initial failures. [4] However, acquiring this information can be too expensive (compared to the alternatives), and the Bellman equation

realizes this in advance. Previous is a practical example of the power of simulating the events and information gained ahead of time. In the real world, design maturity limits the effectiveness of the redesign process [1]. After the redesign, one would have to return to the information state of the fatigue strength and



**Figure 1.** Dynamic optimization: making the decision example.

continue the decision-making with revised fatigue loading. Sometimes optimization of the machine component might not be enough, but a change in material for one with higher fatigue strength is also required. It poses an exciting challenge to the decision-making theory; how much value is there in the material-specific information and manufacturing knowledge?

**Keywords:** Fatigue, dynamic programming, Bellman equation, finite-horizon backward induction.

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