## Three-dimensional solids and structures within strain gradient elasticity – numerical methods and model comparisons

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## Abstract

Literature on the most common dimensionally reduced structural models within different theories of generalized continuum mechanics is vast, most probably due to the relative commonness, simplicity and applicability of the corresponding well-known models of the classical continuum mechanics. The small amount of generalized constitutive parameters incorporated into these reduced models has definitely been an attraction for scholars as well [1]. On the contrary, literature on the three-dimensional formulations of generalized continuum mechanics is still quite limited, partly due to the large number of constitutive parameters incorporated into the models and partly due to the relatively high complexity and computational cost of the related numerical methods [2, 3]. This contribution focuses on comparing the structural dimension reduction models to the corresponding three-dimensional "parent" models of strain gradient elasticity, especially in the context of microarchitectural, lattice or cellular, structures [3, 4]. These comparisons rely on analytical and numerical solutions of the different models and incorporate constitutive parameters obtained by different computational homogenization techniques. For microarchitectural structures, non-homogenized fine-grain models of classical elasticity provide reference solutions, and hence enable a simulation-based model validation. Ritz–Galerkin methods, in the form of higher-order finite element methods, are adopted for obtaining reliable numerical solutions for problems lacking for analytical benchmark solutions. Some of the main characters of the structural models, e.g., the so-called stiffening effect for beams, are in accordance with the three-dimensional models – and with the fine-grain reference models when regarding microarchitectural structures.

Keywords: generalized continuum mechanics, elasticity theory, thin-walled structures, finite element methods, computational homogenization

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