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Mathematical model of rotor: considering a finite element model

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ABSTRACT

A rotor system for applying Active Magnetic Bearings control is considered. A rigid body model is often used to analyze the motion of a rotor. The finite element model is implemented to take into account the flexibility of the rotor and, as a result, the presence of various mode shapes of the system.

When constructing a finite element model, the beam elements are considered according to the Bernoulli theory or the Timoshenko theory [1]. The influence of the use of one of these theories on the frequencies and mode shapes is considered, then the influence on the control will also be considered.

The figure shows one beam element of the rotor [2]. The movement of nodes along the z-axis is not considered, since it occurs only when the rotor speed changes, which would mean the system is non-linear due to the gyroscopic effect.

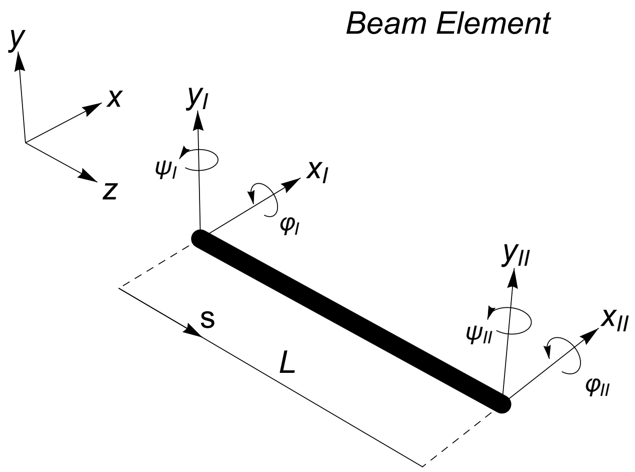

Figure 1. Beam element for FEM model

The connection of the beam elements and, accordingly, the increase in the number of nodes allows for more accurate modelling of a flexible rotor and takes into account more mode shapes.

A program was written in Wolfram Mathematica for finding the matrices of mass, elasticity and gyroscopic effect. A simple method is proposed for further use of the obtained data in modelling in other programs. In particular, this is achieved by carrying out the entire mathematical analysis in a dimensionless form.

$$M - \text{mass matrix},\;\; K - \text{stiffness matrix},\;\; G - \text{gyroscopic matrix}.$$

Matrices M, K, G are compiled for the transition to matrix A for further study of the AMB control:

$$A = \begin{pmatrix} O & I \\ -M^{-1}K & -\Omega M^{-1}G \end{pmatrix}$$

where O – zeros matrix and I – identity matrix.

The advantage of this algorithm for obtaining a mathematical model is the addition of any other elements associated with the rotor (for example, rigid disks), since the equations were written using the Lagrangian and representations for kinetic and potential energies.

The result is represented by various animations and motion graphics of the rotor system. The simulation results are presented with Matlab, Simulink and Wolfram Mathematica software.

Thus, a finite element model of the rotor is presented in the paper for modelling the Active Magnetic Bearings control, taking into account the flexibility of the rotor.

Keywords: AMB rotor, Bernoulli and Timoshenko beams, finite element model, mode shapes.

REFERENCES

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