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A posteriori estimates by the hypercircle method

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ABSTRACT

The classical Prager-Synge hypercircle theorem [4] for linear elasticity can be stated as follows. Let σ be the exact stress field and let Σ be a statically admissible approximation to it. Further, let U be a kinematically admissible approximation to the displacement and let $\mathcal{A}\varepsilon(U)$ be the corresponding stress approximation with $\mathcal{A}$ being the constitutive equation. Then it holds

$$\|\sigma - \frac{1}{2}\big(\Sigma + \mathcal{A}\varepsilon(U)\big)\|_E = \frac{1}{2}\|\Sigma - \mathcal{A}\varepsilon(U))\|_E,$$

where $\|\cdot\|_E$ is the energy norm. In other words, with $\frac{1}{2}(\Sigma + \mathcal{A}\varepsilon(U)$ as the approximation, we know the error exactly.

This theorem we use in the following manner [2, 3]. By a mixed method we obtain a statically admissible stress field. The kinematically admissible displacement we get by a two-step postprocessing of the displacement of the mixed method. By a local, element-by-element, computation [1, 5], we get a new displacement field with an increased accuracy. From this we compute a new continuous displacement by averaging the degrees of freedom along edges and nodes. For the approach we perform a complete numerical analysis and present supporting numerical results.

In addition, we present the analogous approach for the Hellan-Herrmann-Johnson plate bending family.

Keywords: mixed finite finite elements, postprocessing, a posteriori error estimates, hypercircle method

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