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# Coupling terms and surface effects in combined second strain gradient elasticity and plasticity theories

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## ABSTRACT

In the present contribution, we combine Mindlin's second strain gradient elasticity theory [6] and Gurtin-Anand-Gudmundson gradient plasticity theory [1, 2] to form a unified framework [5]. This augments the modelling capabilities by incorporating elasticity-related length scales along with plasticity-related energetic and dissipative ones. The proposed model accounts for coupling terms with respect to elastic strains and second gradient of elastic strains as well as plastic strains and second gradient of plastic strains [4].

The sixth-order boundary value problems of the second strain gradient elasto-plasticity model are formulated in a variational form within an $H^3$ Sobolev space settings. Conforming Galerkin discretizations for numerical results are obtained via an isogeometric approach [3]. For a simple shear problem of a strip, it is shown that the elasticity-related and plasticity-related (dissipative and energetic) coupling length scales augment the modelling capabilities by capturing elastic, dissipative (yield) and energetic (strain) softening phenomena.

**Keywords:** size effects, surface effects, coupling terms, lattice structures, isogeometric analysis.

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